Engineering Notes

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Relative Rotary Motion of Liquids in Rotating Cylinders under Large Amplitude Axial Vibrations

N. F. Eraslan*
Louisiana State University, Baton Rouge, La.

Nomenclature

D = inner diameter of container, in.
d = displacement (twice the amplitude), in.
f = frequency of axial vibration, cps
h_w = height of liquid (water) in the container at rest, in.
n_c = container rpm; relative to a fixed system of reference
n_w = mean rpm of the liquid (water); relative to the fixed system of reference

Introduction

THE motion of a liquid under axial excitations in a partially filled, vertical cylindrical container was discussed by Stokes[1] as early as 1847. He reasoned that since the forward motion of the particles was not altogether compensated for by their backward motion, the liquid should necessarily have a net angular momentum along the direction of rotary sloshing of the free surface.

An extensive literature survey on analytical investigations of the mathematical problems associated with rotary sloshing and solutions under various simplifying approximations is given by Habib.[2] Theoretical and experimental investigations by several authors, among them Hutton,[3] have shown that, under certain conditions, the approximate mathematical solutions that quantitatively predict the behavior of the rotary sloshing and the associated angular momentum are in marked discrepancies with the experimental data. Consequently, it is essential to carry out experimental investigations to determine the conditions under which the simplified theoretical analyses can not be considered valid.

The most extensively studied mode of sloshing of liquids in vertical cylindrical containers deals with theoretical and experimental investigations under the conditions of moderate angular velocities and high-frequency, small-amplitude axial vibrations. This Note presents the results of a purely experimental investigation of the relative motion of liquids in vertical cylindrical containers under the influence of substantial angular velocities and low-frequency, large amplitude axial vibrations.

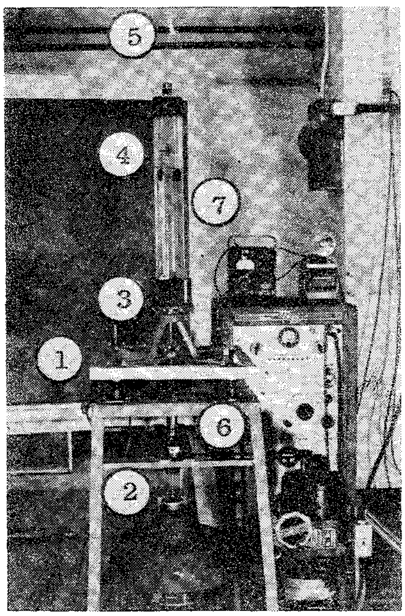

Fig. 1 Experimental apparatus.

Experimental Apparatus

The experimental apparatus shown in Fig. 1 consists of the following units as indicated in the figure: 1) A shaking table of four vertical "sleeve-rod" type guiding assemblies and four helical springs. 2) A "slotted sleeve-peg" type coupling which allows simultaneous rotational and axial motions of the container. 3) A spinner assembly to measure the mean angular velocity of the water in the container. The two vertical paddles at the end of the fork shaped spinner are immersed into the liquid until their lower edges become 2 in. below the horizontal free surface. Further immersion results in disturbances in the relative motion of the liquid with possibilities of the destruction of the free vortex motion. (The paddles were $\frac{1}{16}$-in. × 1.0-in. × 1.5-in. aluminum plates. Clearances between paddles and walls of the small and large containers were $\frac{1}{2}$ in. and 1.0 in., respectively. At these "reference" positions the paddles were in the most stable region of the vortex and far enough from the surface of the core, which is easily disturbed by the close presence of the solid pieces. Because of the relative size of the paddles, the measured rpms represent a "mean" value rather than a "punctual" one.) 4) A white colored pointer for stroboscopic observations of the relative motion. The pointer is connected to the spinner by a vertical rod carried by two ball bearings. 5) Containers made of transparent acrylic cylinders.

The following standard instruments and equipment also were used: a strobotac, a vibration meter, a speed variator, a "pulley-vee belt" assembly, and a 50-lb capacity shaker.

Experimental Procedure and Qualitative Observations

The main part of the experimental investigation was conducted with water as the single liquid. The experiments were carried out with two different size containers of 3.5- and 8.2-in. i.d. Water heights in the containers were varied between 13 in. and 18 in. The angular velocity of the container was varied between 95 and 325 rpm, the frequency between 5 and 8 cps, and the amplitude of the axial vibrations between 0 and 0.5 in.

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* Professor of Mechanical Engineering, Department of Mechanical, Aerospace, and Industrial Engineering.

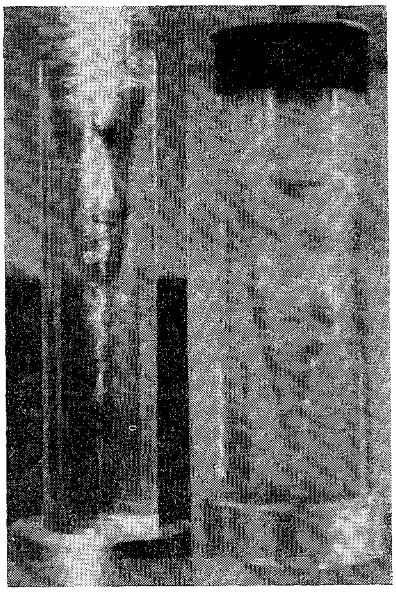

Fig. 2 Fully developed vortex cores.

Fig. 4 Formation of two vortices for large amplitude vibrations.

For a specified set of water height h_w, frequency f, and angular velocity n_c, the onset of relative motion between the water and the container depended on a certain magnitude of the amplitude of the vibration. Further amplitude increases resulted in substantial increases in the relative angular motion.

Since the relative motion of the liquid with respect to the container, in a sense, represented a "free vortex" type of motion, the formation of axisymmetric cores was expected. Such free vortices were observed earlier and reported in Refs. 4 and 5. Under steady-state excitations, the cores develop into their steady-state, axisymmetric, conical shapes and approximately retain these shapes as long as the uniform conditions exist. Figures 2a and 2b show these fully developed cores for 3.5- and 8.2-in. containers.

Figure 3 presents experimental variations of the mean relative angular velocity of the liquid with the ratio of the amplitude of the vibrations to the water height in the container, parametrically with frequency f and various specified combinations of the height of the water h_w and the rotational velocity n_c of the container. One important characteristic is that, in general, the mean relative velocity increases with the amplitude of the vibrations. For certain conditions, the maximum mean relative velocity reaches approximately 110% of the angular velocity of the container.

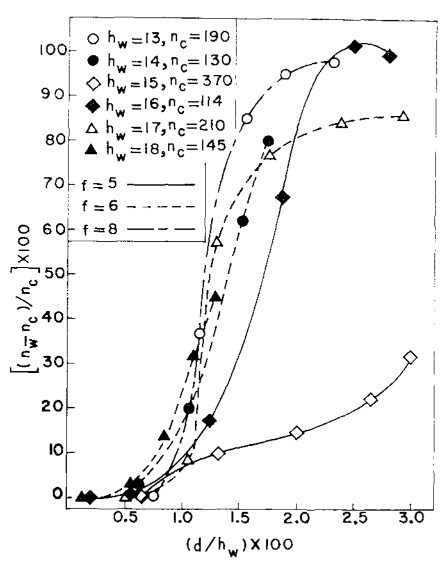

Fig. 3 Variations of mean relative angular velocity with height of water.

In most cases, the free vortex cores were axisymmetric about the axis of the container, and they pulsated slightly with a certain frequency along the axis. However, under special cases, e.g., $D = 8.2$, $h_w = 14$, $f = 5$, $n_c = 115$, and $d = 0.46$, the cores moved around in a rather erratic fashion similar to the motion of the tornadoes in nature.

Experiments with Two Different Liquids

As an extension of the experiments with a single liquid, the motion of two immiscible liquids, water and gasoil, with distinctly different densities and viscosities, was investigated under the influence of axial vibrations in a rotating, vertical cylinder.

For vibrations with relatively small amplitudes, the relative motions of the liquids result in two axisymmetric vor-

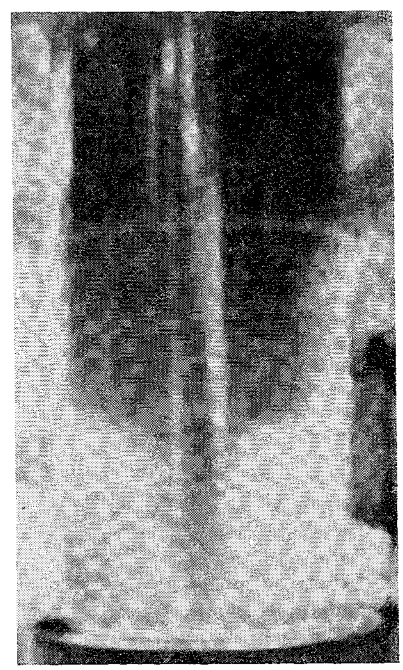

Fig. 5 Filling of the lower core with lower-density liquid.

tices with approximately conical shapes in the two different liquid layers inside the container (Fig. 4). The vortex core in the lower-density liquid (gasoil) of the upper layer contains an air-filled core, similar to cases with a single liquid with the apex of the cone downward. However, in the higher-density liquid (water) of the lower layer, the vortex core reverses orientation and the apexes of the two cones meet at a point between the liquid layers with the lower core completely filled with the higher-density liquid of the lower layer. As the amplitude of the axial vibration is increased, the upper core eventually pierces into the lower one and destroys the inverted shape of the lower vortex core. Then the system contains only one vortex core, which is completely filled with air, similar to the cases with a single liquid.

If the large amplitude axial vibration is suddenly terminated, the lower portion of the core in the water layer immediately fills with the lower-density liquid of the upper layer, resulting in a single core filled with the lower-density liquid (Fig. 5). If the axial vibration is restarted during this period, the lower portion of the vortex core becomes immediately cleared of the low-density liquid, and the relative motion retains the conditions when a single air-filled vortex core exists for both liquid layers.

References

[1] Stokes, G. G., "On the Theory of Oscillatory Waves," *Cambridge Philosophical Society*, Vol. 8, 1847, pp. 441–445.

[3] Habib, L. M., "On the Mechanics of Liquids in Subgravity," *Astronautica Acta*, Vol. II, No. 6, Nov.–Dec., 1965, pp. 401–409.

[3] Hutton, R. E., "Fluid-Particle Motion during Rotary Sloshing," *Journal of Applied Mechanics*, March 1964, pp. 123–130.

[4] Elshinnawy, A. M., "Surface Waves on a Vortex," PhD thesis, 1965, Columbia Univ.

[5] Lee, C. H., "Analytical and Experimental Investigations on the Free Surface Oscillations of Rotating Liquids," MS thesis, Jan. 1966, Dept. of Mechanical and Aerospace Engineering, Louisiana State Univ.